Package 'FDGcopulas'

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Type Package

Title Multivariate Dependence with FDG Copulas

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Description FDG copulas are a class of copulas featuring an interesting balance between flexibility and tractability. This package provides tools to construct, calculate the pairwise dependence coefficients of, simulate from, and fit FDG copulas. The acronym FDG stands for 'one-Factor with Durante Generators', as an FDG copula is a one-factor copula -- that is, the variables are independent given a latent factor -- whose linking copulas belong to the Durante class of bivariate copulas (also referred to as exchangeable Marshall-Olkin or semilinear copulas).

License GPL (>= 3)

Depends Rcpp (>= 0.10.6), methods

LinkingTo Rcpp

Imports numDeriv, randtoolbox

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`FDGcopulas-package` *Deals with FDG copulas*

Description

Constructs, simulates from, calculates dependence coefficients, and fits FDG copulas.

Details

Package:	FDGcopulas
Type:	Package
Version:	1.0
Date:	2014-09-19
License:	GPL >=3

See the examples below to have an overview of how what the package can offer.

Note

Feel free to contact the authors if have any comments, suggestions, or want to report a bug.

Author(s)

Gildas Mazo Stephane Girard Maintainer: Gildas Mazo <gildas.mazo@free.fr>

References

Mazo G., Girard, S., Forbes, F. A flexible and tractable class of one-factor copulas, http://hal.archives-ouvertes.fr/hal-00979147

Examples

```
## creates an object of class 'FDGcopula'
myFDGcopula <- FDGcopula("frechet", c(.3,.5,.7,.9))

## compute the pairwise dependence coefficients
## Spearman's rho:
rhoFDG(myFDGcopula)
## Kendall's tau:
tauFDG(myFDGcopula)
## Upper tail dependence coefficient:
```

```
utdcFDG(myFDGcopula)
## Lower tail dependence coefficient:
ltdcFDG(myFDGcopula)

## simulates data ##
dat <- rFDG(30, myFDGcopula)

## fit data ##
myFittedCopula <- fitFDG(myFDGcopula, dat)
```

---

corFDG *Sample pairwise dependence coefficients*

Description

Computes sample (i.e., empirical) pairwise dependence coefficients such as the Kendall's tau, the Spearman's rho, and the upper tail dependence coefficient (the later being proved to have good statistical properties only for extreme-value copulas and in the case of known margins).

Usage

```
corFDG(x, depcoefType)
```

Arguments

x	the data
depcoefType	type of dependence coefficient: one of 'kendall', 'spearman', or 'utdc'

Details

The function 'corFDG' simply calls 'cor()' when 'depcoefType' is either 'kendall' or 'spearman'. If it is 'utdc', then the sample upper tail dependence coefficient for extreme-value copulas introduced in the reference below is computed.

Value

A matrix of size 'd' times 'd' where 'd' is the number of variables. The element in the i-th row and j-th column is the dependence coefficient between the i-th and j-th variable.

Author(s)

Gildas Mazo

References

Ferreira, M., Nonparametric estimation of the tail-dependence coefficient, REVSTAT–Statistical Journal, 2013

See Also

cor

Examples

```
## Generate an extreme-value copula
myFDGcopula <- FDGcopula("frechet", c(.3,.5,.7,.9), extremevalue=TRUE)
dat <- rFDG(100, myFDGcopula)
## Compute the sample upper tail dependence coefficient given in the reference
corFDG(dat, "utdc")
```

FDGcopula *Construction of FDG copula class object*

Description

Constructs a FDG copula class object.

Usage

```
FDGcopula(family, parameters, extremevalue=FALSE, checkbounds=TRUE)
```

Arguments

family	character to indicate the family of the generators
parameters	the parameter vector
extremevalue	boolean indicating if the extreme-value limit copula corresponding to the FDG copula is to be constructed instead
checkbounds	boolean indicating if an error message is to be produced in case of parameter values are out of their theoretical bounds

Details

The possible families are 'cuadrasauge', 'frechet', 'sinus' and 'exponential'. Control over the possibility of NOT returning an error message in case of wrong parameters is given to be able to use certain optimization algorithms.

Value

An object of class FDGcopula-class

Author(s)

Gildas Mazo

References

Mazo G., Girard, S., Forbes, F. A flexible and tractable class of one-factor copulas, http://hal.archives-ouvertes.fr/hal-00979147

See Also

FDGcopula-class for a description of the class

Examples

```
## Create a FDG copula with Cuadras-Aug\'e generators
## and parameter vector 'theta'
theta <- c(.3,.5,.7,.9)
myFDGcopula <- FDGcopula("cuadrasauge", theta)
str(myFDGcopula)

## Create its extreme-value limit copula
myExtremeValueFDGcopula <- FDGcopula("cuadrasauge", theta,
extremevalue=TRUE)
str(myExtremeValueFDGcopula)

## Note: the two above copulas are NOT the same!
```

FDGcopula-class *Class* "FDGcopula"

Description

Class to represent a FDG copula

Slots

dimension: Object of class "integer" Dimension of the copula

parameters: Object of class "numeric" Vector of parameters

family: Object of class "character" Family of the generators

extremevalue: Object of class "logical" Boolean indicating if the copula is an extreme-value copula

parameterrange: Object of class "numeric" Interval containing the bounds of the parameters

Methods

No methods defined with class "FDGcopula" in the signature.

Author(s)

Gildas Mazo

References

Mazo G., Girard, S., Forbes, F. A flexible and tractable class of one-factor copulas, http://hal.archives-ouvertes.fr/hal-00979147

See Also

FDGcopula for constructing FDG copulas

Examples

```
showClass("FDGcopula")
```

---

fitFDG *Estimation of FDG copulas*

Description

Estimates FDG copulas.

Usage

```
fitFDG(FDGcopula, data, depcoefType="spearman", nbInit=1,
                   W=NA, method="L-BFGS-B", estimate.variance=TRUE,
                   nb.rep=100, nb.obs=100, dcData=NA, sizeSubSample=10000)
```

Arguments

FDGcopula	copula to fit the data
data	data to be fitted
depcoefType	type of dependence coefficient to be used in the estimation method
nbInit	number of initialisations in the optimization algorithm to find the global optimum
W	weight matrix to be used in the estimation method
method	the 'method' for 'optim()'
estimate.variance	boolean indicating if the asymptotic variance-covariance matrix should be estimated
nb.rep	number of replications to be used in the estimation of the asymptotic variance-covariance matrix (it has no effect if estimate.variance=FALSE)
nb.obs	size of the simulated samples on which is based the estimation of the asymptotic variance-covariance matrix
dcData	if not NA, matrix of size 'd' times 'd', where 'd' is the dimension of the fitted copula, consisting of the sample pairwise dependence coefficients that the user wants to use for the estimation method
sizeSubSample	size of the sample over which is to be taken the maximum when generating extreme-value copulas (needed if estimate.variance=TRUE and FDGcopula@extremevalue=TRUE, no effect otherwise)

Details

The method used to estimate the parameters of FDG copulas is a weighted least squares estimator based on dependence coefficients (see [2]). The coefficients implemented are the Spearman's rho, the Kendall's tau, and the upper tail dependence coefficient in the case of extreme-value copulas. If the user wishes to use other coefficients, it is possible but he/she should provide his own sample pairwise dependence coefficients with the matrix `dcData`. The estimation of the asymptotic variance-covariance matrix of 'sqrt(n)(theta hat - theta)', where 'n' is the sample size, 'theta' is the parameter vector, and 'theta hat' is the weighted least square estimator, is carried out by simulation. More precisely, `nb.rep` replications of datasets of size `nb.obs` are simulated according to the fitted FDG copula. For each dataset, the sample dependence coefficients are calculated, and, then, their sample variances / covariances are computed. In the case where the upper tail dependence coefficients were chosen to perform the estimation, a different approximation is used. Since the margins are assumed to be known, there is a simple formula for the variances / covariances given in (15) of [2]. These quantities within this formula can be approximated by standard empirical means calculated on a single big dataset from the underlying extreme-value copula. To simulate that dataset, the variable `sizeSubSample` is used along with `nb.rep`: `nb.rep` sub-samples of size `sizeSubSample` are simulated, and for each sub-sample, the maximum is taken, thus leading to a final dataset of size `nb.rep`. The empirical means to approximate the asymptotic variances / covariances are computed on this last final dataset.

Value

A `fitFDG` class object containing the slots:

`estimate`	the estimated parameter vector
`var.est`	the asymptotic variance-covariance matrix
`optimalValues`	the optimal value(s) of the loss function
`convergence`	the output monitor parameters returned by 'optim()'
`copula`	an object of the same class as FDGcopula, where the slot containing the parameters is filled with the estimated parameters

Author(s)

Gildas Mazo

References

[1] Mazo G., Girard, S., Forbes, F. A flexible and tractable class of one-factor copulas, http://hal.archives-ouvertes.fr/hal-00979147

[2] Mazo G., Girard, S., Forbes, F. Weighted least-squares inference based on dependence coefficients for multivariate copulas, http://hal.archives-ouvertes.fr/hal-00979151

See Also

`fitFDG-class`

Examples

```
## Create an object of class 'FDGcopula'
theta <- c(.3,.5,.7,.9)
myFDGcopula <- FDGcopula("frechet", theta)

## Generate a sample from a FDG copula with Frechet generators
## and parameter vector 'theta'
dat <- rFDG(100, myFDGcopula)
## Fit a FDG copula to the data
myFittedCopula <- fitFDG(myFDGcopula, dat)
myFittedCopula
```

---

fitFDG-class *Class* "fitFDG"

Description

Class to represent fitted FDG copulas

Slots

estimate: Object of class "numeric" estimated parameter vector

var.est: Object of class "matrix" estimated variance-covariance matrix of scaled estimator

optimalvalues: Object of class "numeric" optimal values of the loss function which was minimized during the estimation procedure

convergence: Object of class "list" monitoring parameters returned by 'optim()'

FDGcopula: Object of class "FDGcopula" estimated copula

Methods

No methods defined with class "fitFDG" in the signature.

Details

The estimated variance-covariance matrix is that of the estimator times the square root of the sample size.

Author(s)

Gildas Mazo

References

[1] Mazo G., Girard, S., Forbes, F. A flexible and tractable class of one-factor copulas, http://hal.archives-ouvertes.fr/hal-00979147

[2] Mazo G., Girard, S., Forbes, F. Weighted least-squares inference based on dependence coefficients for multivariate copulas, http://hal.archives-ouvertes.fr/hal-00979151

See Also

fitFDG for fitting FDG copulas

Examples

```
showClass("fitFDG")
```

---

ltdcFDG *Lower tail dependence coefficient of FDG copulas*

Description

Calculates the lower tail dependence coefficient of FDG copulas.

Usage

```
ltdcFDG(FDGcopula)
```

Arguments

FDGcopula	the FDG copula class object

Value

A matrix of size 'd' times 'd', where 'd' is the dimension of the copula. The element in the i-th row and j-th column is the dependence coefficient of the i-th and j-th variable.

Author(s)

Gildas Mazo

References

Mazo G., Girard, S., Forbes, F. A flexible and tractable class of one-factor copulas, http://hal.archives-ouvertes.fr/hal-00979147

See Also

rhoFDG, tauFDG, utdcFDG

Examples

```
## FDG copula with 'exponential' generators
myExpo <- FDGcopula("exponential", 1:4)
ltdcFDG(myExpo)

## FDG copula with Cuadras-Auge generators
myCA <- FDGcopula("cuadrasauge", c(.3,.5,.7,.9))
ltdcFDG(myCA) # lower tail independent
```

rFDG *Simulation of FDG copula models*

Description

Simulates from FDG copula models.

Usage

```
rFDG(n, FDGcopula, sizeSubSample=10000)
```

Arguments

n	size of the sample to be generated
FDGcopula	copula from which the sample is to be generated
sizeSubSample	size of the sub-samples over which is to be taken the maximum when generating extreme-value copulas

Details

An observation of an extreme-value distribution is essentially the maximum over a sub-sample of (theoretically) infinite size. In practice, the size of this sub-sample is set to sizeSubSample.

Value

A matrix of size 'n' times the dimension of the copula.

Author(s)

Gildas Mazo

References

Mazo G., Girard, S., Forbes, F. A flexible and tractable class of one-factor copulas, http://hal.archives-ouvertes.fr/hal-00979147

Examples

```
## Generate a sample from a FDG copula with Fr\'echet generators
## and parameter vector 'theta'
theta <- c(.3,.5,.7,.9)
myFDGcopula <- FDGcopula("frechet", theta)
dat <- rFDG(30, myFDGcopula)
head(dat)
```

rhoFDG *Spearman's rho of FDG copulas*

Description

Calculates the Spearman's rho of FDG copulas.

Usage

```
rhoFDG(FDGcopula)
```

Arguments

FDGcopula	the FDG copula class object

Value

A matrix of size 'd' times 'd', where 'd' is the dimension of the copula. The element in the i-th row and j-th column is the dependence coefficient of the i-th and j-th variable.

Author(s)

Gildas Mazo

References

Mazo G., Girard, S., Forbes, F. A flexible and tractable class of one-factor copulas, http://hal.archives-ouvertes.fr/hal-00979147

See Also

tauFDG, utdcFDG, ltdcFDG

Examples

```
## FDG copula with Frechet generators
myFDGcopula <- FDGcopula("frechet", c(.3,.5,.7,.9))
rhoFDG(myFDGcopula)
```

tauFDG *Kendall's tau of FDG copulas*

Description

Calculates the Kendall's tau of FDG copulas.

Usage

```
tauFDG(FDGcopula)
```

Arguments

FDGcopula	the FDG copula class object

Value

A matrix of size 'd' times 'd', where 'd' is the dimension of the copula. The element in the i-th row and j-th column is the dependence coefficient of the i-th and j-th variable.

Author(s)

Gildas Mazo

References

Mazo G., Girard, S., Forbes, F. A flexible and tractable class of one-factor copulas, http://hal.archives-ouvertes.fr/hal-00979147

See Also

rhoFDG, utdcFDG, ltdcFDG

Examples

```
## FDG copula with Frechet generators
myFDGcopula <- FDGcopula("cuadrasauge", c(.3,.5,.7,.9))
tauFDG(myFDGcopula)
```

utdcFDG *Upper tail dependence coefficient of FDG copulas*

Description

Calculates the upper tail dependence coefficient of FDG copulas.

Usage

```
utdcFDG(FDGcopula)
```

Arguments

FDGcopula	the FDG copula class object

Value

A matrix of size 'd' times 'd', where 'd' is the dimension of the copula. The element in the i-th row and j-th column is the dependence coefficient of the i-th and j-th variable.

Author(s)

Gildas Mazo

References

Mazo G., Girard, S., Forbes, F. A flexible and tractable class of one-factor copulas, http://hal.archives-ouvertes.fr/hal-00979147

See Also

rhoFDG, tauFDG, ltdcFDG

Examples

```
## FDG copula with 'exponential' generators
myFDGcopula <- FDGcopula("exponential", 1:4)
utdcFDG(myFDGcopula) # upper tail independent
## FDG copula with 'sinus' generators
mySinus <- FDGcopula("sinus", c(.9,1.2,1.3,1.55))
utdcFDG(mySinus)
```

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